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Lifting-Surface Theory for Skewed and Swept Subsonic Wings

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A new method is developed for solving the lifting-surface equation for thin subsonic wings which can also be interpreted as a vortex lattice method. The downwash equation is transformed into double integrals involving Cauchy-type singularities in the chordwise and spanwise directions. A technique developed by Lan for airfoil theory is used to reduce both integrals to a double summation. This method properly accounts for the leading-edge singularity, Cauchy singularity, and Kutta condition. The solutions generally compared well with other lifting-surface theories, but with much smaller computational times, and the method was found to be more accurate and converge faster than conventional vortex lattice methods.

Nomenclature

A_{kn} = unknowns in the downwash matrix
AR = aspect ratio
b = wing span
c = wing chord
$\bar{c}$ = mean chord of wing
c_R = wing root chord
c_T = wing-tip chord
c_{TL} = left wing-tip chord
c_{TR} = right wing-tip chord
C_ℓ = sectional lift coefficient
C_L = wing lift coefficient
C_{L_α} = lift curve slope, per degree or per radian
C_m = sectional moment coefficient
C_M = wing pitching moment coefficient, about leading edge at midspan
m_{LE} = $\tan\Lambda_{LE}$
m_{TE} = $\tan\Lambda_{TE}$
Δm = $\tan\Lambda_{TE} - \tan\Lambda_{LE}$
M = number of spanwise vortices
n = summational integer
N = number of chordwise vortices and control points
S = wing planform area
VLM = vortex lattice method
V_∞ = freestream velocity
w = downwash velocity, positive upward
w_L = downwash velocity induced by the vortices on left wing panel
w_{LC} = downwash velocity induced by the left vortex filament at the wing midspan
w_R = downwash velocity induced by the vortices on right wing panel
w_{RC} = downwash velocity induced by the right vortex filament at the wing midspan
x = chordwise coordinate measured from leading edge in direction of V_∞
x_{ac} = wing aerodynamic center location
y = spanwise coordinate on right wing panel, positive to the right
$\bar{y}$ = spanwise coordinate on left wing panel
z_c = vertical coordinate of mean camber line
α = angle of attack, deg
γ = nondimensional circulation per unit chord
Γ = circulation
η = nondimensional chordwise coordinate, see Eq. (2)
θ = transformed chordwise coordinate
λ = wing taper ratio
Λ_{LE} = leading-edge sweep, deg
Λ_{TE} = trailing-edge sweep, deg
ϕ = transformed spanwise coordinate on right wing panel
$\bar{\phi}$ = transformed spanwise coordinate on left wing panel

Subscripts

i = chordwise control point, see Eq. (8)
j = spanwise control point, see Eq. (9)
k = chordwise vortex point, see Eq. (10)
ℓ = spanwise vortex point, see Eq. (11)
LE = leading edge of wing
TE = trailing edge of wing

Introduction

THE motivation for the research described in this paper stemmed from an experimental investigation of a wing-rotor interaction system for helicopters. A complete description of this investigation is given in Ref. 1, and a condensed version appears in Ref. 2.

The test results showed significant increases in lift and decreases in drag for both the wing and the proprotor due to favorable interference effects. This and other features made the concept attractive for future design considerations and prompted the present analytical investigation of subsonic wings.

An analytical investigation of a complete wing-rotor flowfield requires computational methods for both the wing and the rotor. In order to calculate the combined flowfield efficiently, it is necessary to have a method which will calculate subsonic wing properties accurately and expeditiously. It is the purpose of this paper to develop such a method.

Aerodynamic characteristics of wings at subsonic speeds are generally calculated by vortex lattice or lifting-surface methods. Vortex-lattice methods are simplier and easier to apply to complex configurations than lifting-surface methods, but they are generally less accurate.[3] A complete description of the vortex-lattice method (VLM) is given in Ref. 4. In lifting-surface theories, a continuous loading or circulation in both the spanwise and chordwise directions is used to determine the aerodynamic characteristics of the wing. Lifting-surface theories are discussed in Ref. 5.

Lan[3] developed an ingenious method for thin, two-dimensional airfoils by using the midpoint trapezoidal rule and the theory of Chebychev polynomials to reduce the

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downwash integral to a finite sum. This method is simple to apply and gives an accurate solution to the airfoil downwash equation. Lan also developed a quasivortex lattice method for finite wings by using his airfoil method for a continuous chordwise vortex distribution in the spanwise direction. The results showed an improvement over the conventional vortex-lattice method. An additional improvement was found empirically by Hough[6] when the lattice was inset one-fourth of a lattice width at the wing tips.

DeJarnette[7] developed a new lifting-surface theory for rectangular wings which used Lan's continuous chordwise vortex distribution but, unlike Lan, a continuous spanwise vortex distribution also. Although the vortex distributions are continuous, the method is easily interpreted as a vortex-lattice method in which the arrangement of horseshoe vortices and control points are determined from the solution, rather than chosen like the conventional vortex-lattice method. It properly locates the tip vortices to be inset from the wing tips, as Hough[6] found empirically. The results were found to compare very closely with other lifting-surface theories, but the computational time was only about one-tenth of the other methods. The results were more accurate and converged much more rapidly than the vortex-lattice method of Ref. 4. A description of the method used for extracting the leading-edge suction parameter from the solution is given in Ref. 8.

The lifting-surface theory given in Refs. 7 and 8 apply the downwash integral in the form involving Cauchy singularities rather than the Mangler singularity form which is usually used in lifting-surface theories. Consequently, difficulties are encountered when the same method is applied to nonrectangular planforms. This paper describes a generalized lifting-surface theory which can be applied to wings which are skewed, swept, or tapered, yet it retains the same accuracy and simplicity as the method given in Refs. 7 and 8. A more detailed description of the developmental procedure is given in Ref. 9.

Lan's Numerical Method

Lan[3] developed an ingenious numerical method for solving the two-dimensional airfoil problem. The midpoint trapezoidal rule is used to reduce the Cauchy form of the downwash integral to a finite summation and the theory of Chebychev polynomials is used to define the vortex and control point positions so that the exact lift, pitching moment, and leading-edge suction are obtained for the flat-plate airfoil. The vortex and control point positions are determined from the solution and then are defined by the "semicircle method," as shown in Fig. 1 for $N=3$. Unlike the conventional vortex-lattice method, the Cauchy singularity, the leading-edge square root singularity, and the Kutta condition are properly accounted for in this method. Accurate results were obtained for airfoils with only a few control points and, thus, this numerical procedure was chosen for the finite wing.

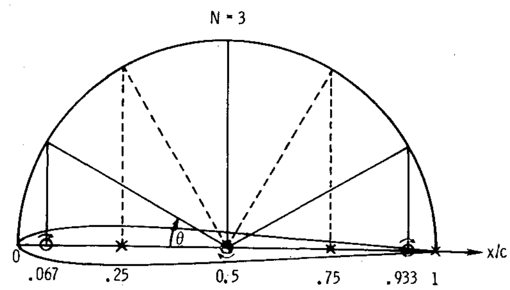
The finite wing problem, however, requires that Lan's method be applied in both the chordwise and spanwise directions on the wing.

Lifting-Surface Theory for Skewed Wings

The downwash equation from lifting-surface theory,[10] in the Mangler integral form, is

$$w(x,y) = \frac{1}{4\pi} \iint_S \frac{\gamma(x_I,y_I)}{(y-y_I)^2} \times \left[1+\frac{x-x_I}{\sqrt{(x-x_I)^2+(y-y_I)^2}}\right] \mathrm{d}x_I \mathrm{d}y_I \tag{1}$$

Equation (1) can be generalized to include skewed wings with taper by transforming the independent variables from x, y to



$x/c = (1 - \cos\theta)/2$

o Vortex Position: $\theta_k = \frac{(2k-1)}{2N}$ (k = 1, ..., N)

X Control Points: $\theta_i = \frac{i\pi}{N}$ (i = 1, ..., N)

Fig. 1 Lan's vortex arrangement for airfoils.

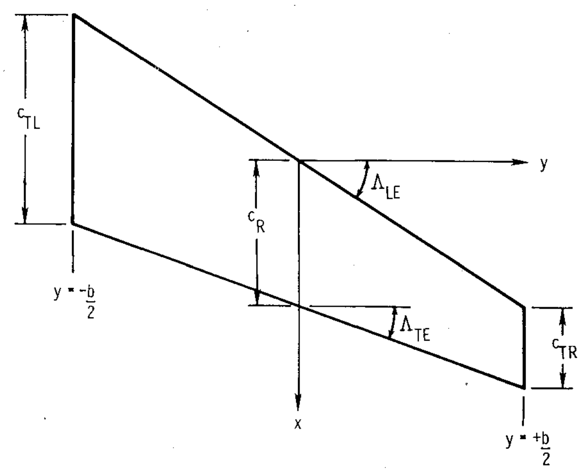


Fig. 2 Geometry and coordinate system for skewed wing.

η, y, where

$$\eta(x,y) \equiv \frac{x - x_{LE}(y)}{c(y)} \tag{2}$$

is the nondimensional chordwise variable $(0 \le \eta \le 1)$. Thus the downwash equation becomes

$$w(\eta,y) = \frac{1}{4\pi} \int\int_S \frac{\gamma(\eta_I,y_I)}{(y-y_I)^2} c(y_I) \times \left[1+\frac{x-x_I}{\sqrt{(x-x_I)^2+(y-y_I)^2}}\right] \mathrm{d}\eta_I \mathrm{d}y_I \tag{3}$$

where

$$x = x_{LE}(y) + \eta c(y)$$

and

$$x_{LE}(y) = y m_{LE} \qquad c(y) = y\Delta m + c_R$$

The geometry and coordinate system of the skewed wing is shown in Fig. 2.

Equation (3) is not a Cauchy-type integral due to the term $(y-y_I)^2$ in the denominator. To apply Lan's numerical method, Eq. (3) is transformed into a Cauchy-type integral form by integration by parts, and with $y = -(b/2)\cos\phi$ and $\eta = (1-\cos\theta)/2$, both integrals are replaced by the midpoint trapezoidal rule sum to get

$$w_{i,j} \simeq \frac{-1}{4\pi \mathit{AR}} \frac{\pi}{M} \frac{\pi}{N} \sum_{\ell=1}^{M} \sum_{k=1}^{N} \left(\frac{\partial(\gamma c/c_R)}{\partial\phi_I}\right)_{k,\ell} \frac{K_{ijk\ell}\sin\theta_k}{\cos\phi_\ell - \cos\phi_j} \tag{4}$$

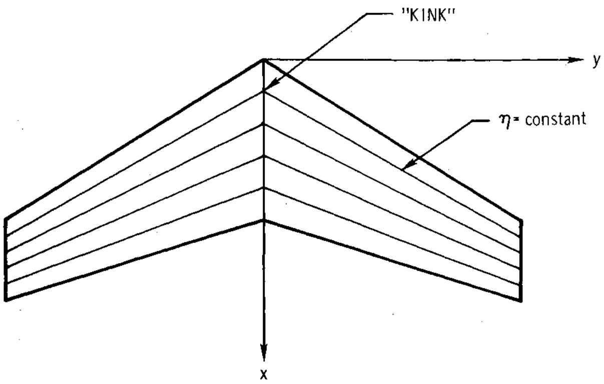


Fig. 3 Spanwise vortex line geometry for swept wing.

where

$$K_{ijk\ell} \equiv 1 + [(A_{sk}^2+1)(\cos\phi_\ell - \cos\phi_j)^2 + 2A_{sk}B_{sk}(\cos\phi_\ell - \cos\phi_j) + B_{sk}^2]^{1/2}/B_{sk} \tag{5}$$

and

$$A_{sk} = m_{\text{LE}} + (\Delta m/2)(1-\cos\theta_k) \tag{6}$$

$$B_{sk} = (\cos\theta_k - \cos\theta_i)\left(\frac{1}{AR} - \frac{\Delta m}{2}\cos\phi_j\right) \tag{7}$$

$$AR = \frac{b}{c_R}, \quad \Delta m = \frac{2(\lambda-1)}{AR(\lambda+1)}, \quad \text{and } \lambda = \frac{c_{TR}}{c_{TL}}$$

The control points are located at

$$\theta_i = i\pi/N \qquad i=1,\ldots,N \quad \text{(chordwise)} \tag{8}$$

and

$$\phi_j = j\pi/M \qquad j=1,\ldots,M-1 \quad \text{(spanwise)} \tag{9}$$

and the integration or vortex points are located at

$$\theta_k = \frac{(2k-1)\pi}{2N} \qquad k=1,\ldots,N\,\text{(chordwise)} \tag{10}$$

$$\phi_\ell = \frac{(2\ell-1)\pi}{2M} \qquad \ell=1,\ldots,M\,\text{(spanwise)} \tag{11}$$

For the skewed wing, M is the number of spanwise vortices across the entire span. The spanwise variation of $\gamma_k(\phi)$ at the chordwise point θ_k is represented by a Fourier sine series

$$\frac{\gamma_k c}{c_R} = \sum_{n=1}^{M-1} A_{kn}\sin n\phi \tag{12}$$

where A_{kn} are unknown parameters. Substitute Eq. (12) into Eq. (4) to get the final form of the downwash as

$$w_{i,j} \simeq \frac{-\pi}{4ARMN}\sum_{\ell=1}^{M}\sum_{k=1}^{N}\sum_{n=1}^{M-1}\frac{A_{kn}n\cos(n\phi_\ell)K_{ijk\ell}\sin\theta_k}{\cos\phi_\ell - \cos\phi_j} \tag{13}$$

The tangent flow boundary condition for thin wings requires that

$$w_{i,j} = \left(\frac{\partial z_c}{\partial x}\right)_{i,j} - \alpha \tag{14}$$

where $z_c(x,y)$ is the shape of the mean camber line. The $N(M-1)$ values of A_{kn} are calculated by applying Eq. (13) at the chordwise and spanwise control points given by Eqs. (8) and (9) and solving the resulting matrix equation. For rectangular wings, only the odd values of n are needed because symmetry properties make the even coefficients zero. However, it is necessary to calculate the A_{kn} for both odd and even values of n for the skewed wing since the spanwise variation of circulation is not symmetric. After the A_{kn} are calculated, the circulation γ_k are determined by Eq. (12). Regardless of the number of chordwise vortices used, there is always a control point at the trailing edge which satisfies the Kutta condition. The sectional and wing aerodynamic characteristics are also calculated by using the midpoint trapezoidal rule to reduce the integrals to finite sums, shown in Ref. 9.

Lifting-Surface Theory for Swept Wings

The downwash equation for tapered wings is given in Eq. (3). It is important to note that Eq. (3) is restricted to planforms with continuous vortex lines in the spanwise direction. For the swept wing, a kink exists in the spanwise vortex lines across the midspan as shown in Fig. 3; thus Eq. (3) must be applied separately to the right and left wing panel. The total downwash at (η,y) on the swept wing is, therefore, composed of the contribution from both wing panels and is defined by

$$w(\eta,y) = \frac{1}{4\pi}\int_0^1\int_0^{b/2}\frac{\gamma(\eta_1,y_1)}{(y-y_1)^2}c(y_1)\times\left[1+\frac{x-x_1}{\sqrt{(x-x_1)^2+(y-y_1)^2}}\right]\mathrm{d}y_1\mathrm{d}\eta_1 + \frac{1}{4\pi}\int_0^1\int_{-b/2}^{0}\frac{\gamma(\eta_1,\bar{y}_1)}{(y-\bar{y}_1)^2}c(\bar{y}_1)\times\left[1+\frac{x-x_1}{\sqrt{(x-x_1)^2+(y-\bar{y}_1)^2}}\right]\mathrm{d}\bar{y}_1\mathrm{d}\eta_1 \tag{15}$$

where $x=x(\eta,y)$ and $x_1=x_1(\eta_1,y_1)$. Note that y_1 and $\bar{y}_1$ are the spanwise integration points for the right and left wing panels, respectively, and y is the spanwise position of a control point. Owing to symmetry, control points are needed on one wing panel only, and the right panel is used here. For integration purposes, it is convenient to define $y_1 = (b/4)(1-\cos\phi_1)$ and $\bar{y}_1 = (b/4)(\cos\bar{\phi}_1 - 1)$.

A Fourier series solution procedure similar to that used for the skewed wing was attempted for the swept wing. This procedure proved unacceptable because of instabilities which developed in the solution as the number of spanwise control points was increased. Results from this method are documented in Ref. 9.

A stable solution is obtained by an alternate method which solves directly for the spanwise derivative of circulation at the integration points. Thus the unknown parameters in the matrix equation are defined as

$$A_{k\ell} \equiv \left|\frac{\partial(\gamma c/c_R)}{\partial\phi_\ell}\right|_{\theta_k} \tag{16}$$

and the circulation at midspan is

$$\tau_{k,\phi=0} \simeq -\frac{\pi}{M}\sum_{\ell=1}^{M}A_{k\ell} \tag{17}$$

The matrix equation will now contain $N(M-1)$ equations with NM unknowns. With $N(M-1)$ control points of Eqs. (8) and (9), N more control points are needed to obtain a solution. It is known that $\partial\gamma/\partial y_1$ is finite at $y=0$, thus, since $y_1=(b/4)(1-\cos\phi_1)$,

$$\left[\frac{\partial\gamma}{\partial y_1}\right]_{y=0} = \lim_{\phi_1\to 0}\frac{\partial\gamma}{\partial\phi_1}\frac{\partial\phi_1}{\partial y_1} = \lim_{\phi_1\to 0}\frac{\partial\gamma}{\partial\phi_1}\frac{1}{(b/4)\sin\phi_1} = \text{finite} \tag{18}$$

therefore,

$$\left.\frac{\partial \gamma}{\partial \phi_1}\right|_{\phi_1=0} = 0$$

which gives N more boundary conditions.

Each wing panel is considered separately, where the trapezoidal geometry is defined by

$$\Delta m \equiv \frac{4(\lambda-1)}{\text{Æ}(\lambda+1)}, \quad c_R \equiv \frac{2b}{\text{Æ}(1+\lambda)}, \quad \text{and} \quad \lambda = \frac{c_T}{c_R}$$

The downwash Eq. (15) is transformed to the Cauchy integral form by integration by parts, thus making it applicable to Lan's numerical method.The variables are transformed from η, y to θ, ϕ and the midpoint trapezoidal rule is applied along with Eq. (17) to the Cauchy integral form of the downwash equation to get the downwash contributions from the right wing panel

$$w_{RC_{i,j}} = \frac{\pi}{\text{Æ}(1+\lambda)NM} \sum_{k=1}^{N} \sum_{\ell=1}^{M} \frac{A_{k\ell}\sin\theta_k}{(1-\cos\phi_j)} K_{RCijk} \tag{19}$$

where

$$K_{RCijk} \equiv 1 + [\,\tfrac{1}{4}(A_1^2+1)(1-\cos\phi_j)^2 + A_1B_1(1-\cos\phi_j) + B_1^2\,]^{1/2}/B_1 \tag{20}$$

and

$$w_{R_{i,j}} = \frac{-\pi}{\text{Æ}(1+\lambda)NM} \sum_{\ell=1}^{M} \sum_{k=1}^{N} \frac{A_{k\ell}\sin\theta_k}{\cos\phi_\ell - \cos\phi_j} K_{R_{ijk\ell}} \tag{21}$$

with

$$K_{R_{ijk\ell}} \equiv 1 + [\,\tfrac{1}{4}(A_1^2+1)(\cos\phi_\ell - \cos\phi_j)^2 + A_1B_1(\cos\phi_\ell - \cos\phi_j) + B_1^2\,]^{1/2}/B_1 \tag{22}$$

and

$$A_1 = m_{\text{LE}} + (\Delta m/2)(1-\cos\theta_k) \tag{23}$$

$$B_1 = (\cos\theta_k - \cos\theta_i)\left[\frac{2}{\text{Æ}(1+\lambda)} + \frac{\Delta m}{4}(1-\cos\phi_j)\right] \tag{24}$$

The same procedure applied to the left wing panel yields

$$w_{LC_{i,j}} = \frac{-\pi}{\text{Æ}(1+\lambda)NM} \sum_{k=1}^{N} \sum_{\ell=1}^{M} \frac{A_{k\ell}\sin\theta_k}{(1-\cos\phi_j)} K_{LCijk} \tag{25}$$

where

$$K_{LCijk} \equiv 1 + [\,\tfrac{1}{4}(A_2^2+1)(1-\cos\phi_j)^2 + A_2B_2(1-\cos\phi_j) + B_2^2\,]^{1/2}/B_2 \tag{26}$$

and

$$w_{Li,j} = \frac{\pi}{\text{Æ}(1+\lambda)NM} \sum_{\ell=1}^{M} \sum_{k=1}^{N} \frac{A_{k\ell}\sin\theta_k}{(2-\cos\phi_\ell - \cos\phi_j)} K_{Lijk\ell} \tag{27}$$

where

$$K_{Lijk\ell} \equiv 1 + [\,\tfrac{1}{4}(A_2^2+1)(2-\cos\phi_\ell-\cos\phi_j)^2 + A_2B_2(2-\cos\phi_\ell-\cos\phi_j) + B_2^2\,]^{1/2}/B_2 \tag{28}$$

and

$$A_2 = -[m_{\text{LE}} + (\Delta m/2)(1-\cos\theta_k)] \tag{29}$$

$$B_2 = [m_{\text{LE}} + (\Delta m/4)(2-\cos\theta_k - \cos\theta_i)](1-\cos\phi_j) + [2/\text{Æ}(1+\lambda)](\cos\theta_k - \cos\theta_i) \tag{30}$$

The vortex positions are given by Eqs. (10) and (11), where M is the number of spanwise vortices on the semispan. The control point locations are given by Eqs. (8) and (9), but the additional N control points on the centerline are generated by applying Eq. (9) at $j=0$.

The downwash equation for the entire swept wing is then

$$w_{i,j} = w_{RC_{i,j}} + w_{R_{i,j}} + w_{LC_{i,j}} + w_{L_{i,j}} \tag{31}$$

Using the tangent flow boundary condition for thin wings, given in Eq. (14), $N(M-1)$ equations are generated for control points corresponding to $i=1,\ldots,N$ and $j=1,\ldots,M-1$.

For the case where $j=0$, special consideration must be given to Eqs. (19) and (25) because they are singular there. These two equations have the property of

$$\lim_{\phi_j \to 0}(w_{RC_{i,j}} + w_{LC_{i,j}}) = -\infty$$

which is interpreted as two vortices trailing from the kinked wing midspan. Physically, these two vortices have the combined effect of inducing an infinite downwash along the midspan. One way to resolve this problem, as noted in Ref. 11, is to round the leading and trailing edges across the

Table 1 Results for skewed planforms; $\Lambda_{\text{LE}} = 30$ deg, $\lambda = 1.0$, Æ = 1.5 and 1.0

Values of C_ℓ/C_L					
	Æ = 1.5 Present	VLM[4]		Æ = 1.0 Present	VLM[4]
$2y/b$	$N=4$, $M=12$	$N=6$, $M=40$	$2y/b$	$N=4$, $N=12$	$N=6$, $M=40$
0.9659	0.3517	0.4032	0.9659	0.3390	0.3884
0.8660	0.6742	0.6908	0.8660	0.6528	0.6683
0.7071	0.9416	0.9423	0.7071	0.9184	0.9185
0.5000	1.1339	1.1258	0.5000	1.1172	1.1085
0.2588	1.2408	1.2276	0.2588	1.2363	1.2227
0	1.2620	1.2464	0	1.2698	1.2544
−0.2588	1.2032	1.2003	−0.2588	1.2186	1.2042
−0.5000	1.0718	1.0608	−0.5000	1.0880	1.0778
−0.7071	0.8748	0.8719	−0.7071	0.8869	0.8850
−0.8660	0.6209	0.6331	−0.8660	0.6274	0.6406
−0.9659	0.3230	0.3682	−0.9659	0.3252	0.3715
Overall values					
	Present	VLM		Present	VLM
C_{L_α}	1.9629	2.0057	C_{L_α}	1.4399	1.4736
$-C_{M_\alpha}$	0.4120	0.4417	$-C_{M_\alpha}$	0.2581	0.1546
x_{ac}/c	0.2048	0.2202	x_{ac}/c	0.1793	0.1049

midspan, thereby eliminating the kink and consequent singularity. However, it was determined that satisfactory results can be obtained by approximating the aforementioned limit by

$$(w_{RC}+w_{LC})_{i,j=0} \simeq \frac{\pi}{4NM}\sum_{\ell=1}^{M}\sum_{k=1}^{N} A_{k\ell}\delta_i^k \sin\theta_k \times\left[\frac{2[m_{LE}+(\Delta m/2)(1-\cos\theta_i)]}{\cos\theta_k-\cos\theta_i}-\Delta m\right] \tag{32}$$

where

$$\delta_i^k = +1 \quad \text{if } \cos\theta_k - \cos\theta_i > 0$$
$$= -1 \quad \text{if } \cos\theta_k - \cos\theta_i < 0$$

Equation (32) generates N more equations when applied for $i=1,...,N$ and $j=0$. Thus, having NM equations and NM unknowns, the $A_{k\ell}$ are calculated by solving the matrix of NM equations. It is again worth noting that there is always a control point at the trailing edge which satisfies the Kutta condition. The sectional and wing aerodynamic characteristics are also calculated by using the midpoint trapezoidal rule to reduce the integrals to finite sums, as shown in Ref. 9.

Results and Discussion

Skewed Wing

Table 1 gives a comparison of the results of the present method with the conventional VLM[4] for two skewed wings with $\Lambda_{LE}=30$ deg, $\lambda=1.0$, and $AR=1.0$ and 1.5. The vortex lattice method is applied for six chordwise and 40 spanwise integration points. The results for the VLM are linearly interpolated to correspond with the present method results. This table shows that the spanwise variation of the sectional lift coefficient compares reasonably well with the VLM. The lift-curve slope is slightly lower than the VLM although the two are in good agreement. The moment coefficient and aerodynamic center location compares reasonably well with the VLM for the $AR=1.5$ wing, but the comparison is poor for the $AR=1.0$ wing.

Swept Wing

A comparison of convergence properties of the lift-curve slope between the present direct matrix method and the VLM of Ref. 4 for three swept wings with $\Lambda_{LE}=45$ deg, $\lambda=0.5$, and $AR=2.0$, 4.5, and 7.0 is shown in Fig. 4. This figure illustrates the slow convergence of the VLM and rapid convergence of the present method. For all three wings, the present method shows good convergence in lift with as few as two chordwise vortices. However, as aspect ratio increases, more spanwise vortices are needed for convergence, though the number is still considerably less than that required by the VLM. Results are similar for wings with taper ratios of 0 and 1 and are therefore not shown.

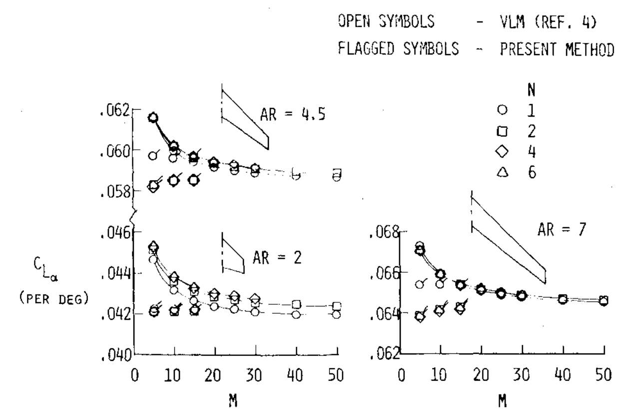


Fig. 4 Convergence comparison of present method with VLM for swept/tapered planforms; $\Lambda_{LE}=45$ deg, $\lambda=0.5$.

Figure 5 compares the spanload distribution between the present method and the VLM of Ref. 4 for a 45-deg swept wing with aspect and taper ratios of 2.0 and 0.5, respectively. With as few as five spanwise vortices, the distribution from the present method compares very well with that from the VLM using 40 spanwise vortices. The accuracy of the present method is primarily attributed to the positions of the vortex and control points, as defined in Eqs. (8-11), which were derived by the theory of Chebychev polynomials. Of additional significance in Fig. 5 is the accuracy with which the sectional lift is predicted at the midspan. This demonstrates that the approach leading to Eq. (32) is a satisfactory treatment of the midspan singularity and that the rounding of the kinked midspan filaments is unnecessary.

Rectangular Wing

The parameters $\Lambda_{LE}=0$ and $\lambda=1$ define the special case of the rectangular wing. For this case, the vortex filaments across the midspan are continuous and thus the singularity there vanishes. A comparison between the present direct matrix method and the method of Ref. 7 is presented in Table 2 for a rectangular wing, $AR=2$. Here, the present method yields a variation of sectional lift coefficient very close to that of Ref. 7. Some difference in the two methods may be as a result of the quadratic interpolation method used to obtain

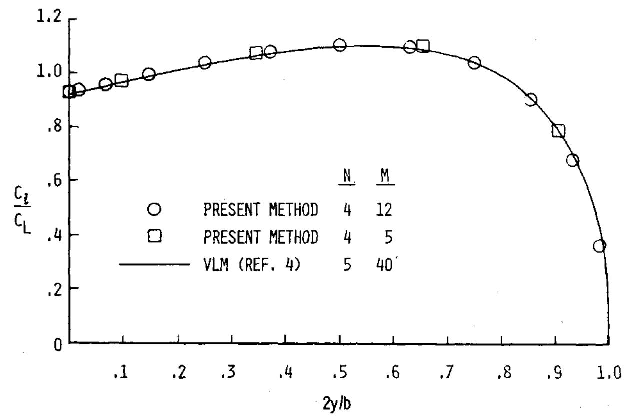


Fig. 5 Comparison of spanload distribution between present method and VLM for swept wing; $\Lambda_{LE}=45$ deg, $AR=2$, and $\lambda=0.5$.

Table 2 Comparison of direct matrix method with method of Ref. 7 for rectangular wing, $AR=2.0$

$2y/b$	Values of C_ℓ/C_L, $N=4$, $M=8$: Direct matrix method (interpolated)	Method of Ref. 7
0	1.2543	1.2543
0.1951	1.2301	1.2335
0.3827	1.1640	1.1706
0.5556	1.0576	1.0651
0.7071	0.9126	0.9171
0.8315	0.7237	0.7290
0.9239	0.4997	0.5084
0.9808	0.2575	0.2618
	Overall values Direct matrix method	Method of Ref. 7
C_{L_α}	2.4732	2.4732
$-C_{M_\alpha}$	0.5187	0.5187
x_{ac}/c	0.2097	0.2097

the comparable sectional lift coefficients. Table 2 also shows that the direct matrix method gives the same overall aerodynamic coefficients as those in Ref. 7. It should be noted that the present method applies the semicircle distribution of control points and integration points over the two half-spans, while the method of Ref. 7 applies the semicircle distribution over the whole span.

All of the results given in this report are for incompressible flow. Subsonic compressibility effects may be included very simply by applying the Prandtl-Glauert rule.[4] The examples computed herein were all performed on an IBM 370/165 computer. Typical cases for the lifting-surface theory required less than 10 s of CPU time.

Conclusions

The following conclusions are drawn from the present investigation:

1) The new lifting-surface method developed herein was found to compare well with other lifting-surface theories, but with much smaller computational times.

2) This new method is more accurate and converges faster than conventional vortex-lattice methods.

3) For the swept wing a direct matrix approach was determined to give more accurate results than the Fourier-sine series approach.

4) The basic equation used in lifting-surface theories gives an infinite downwash at the midspan of swept wings. This singularity in the equation is caused by the "kink" at midspan in the spanwise vortices. A satisfactory treatment of the singularity is to approximate the limit which defines the singularity by a finite summation. Another alternative is to round the leading and trailing edges near midspan, eliminating the kinks in the spanwise vortices.

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